



## **I. BACKGROUND AND QUALIFICATIONS**

1. I graduated from Brown University in 1968. I received a D.Phil. (Ph.D.) in economics in 1973 from Oxford University, where I was a Marshall Scholar. I have been at MIT since completing my D.Phil. My academic specialties are econometrics, the application of statistical methods to economic data, and applied microeconomics, the study of behavior by firms and by consumers.

2. In December 1985, I received the John Bates Clark Award of the American Economic Association, awarded every other year for the most “significant contributions to economics” by an economist under the age of 40. In 1980, I was awarded the Frisch Medal of the Econometric Society. In 2013, I was named a Distinguished Fellow by the American Economic Association. I have been a member of numerous government advisory committees for both the U.S. government and the Commonwealth of Massachusetts. I have published over 170 academic research papers in leading economic journals, including the *American Economic Review*, *Econometrica*, and the *RAND (Bell) Journal of Economics*. I have been an associate editor of *Econometrica*, the leading economics journal, and the *RAND (Bell) Journal of Economics*, the leading journal of applied microeconomics.

3. I am familiar with regression models for the valuation of real estate. I served as an advisor to the Commonwealth of Massachusetts when such models were initially adopted in Massachusetts as the basis for that state’s tax appraisals. I served as the Special Witness (Master) for the Honorable John R. Bartels, U.S. District Court for the Eastern District of New York, during 1981-82. In that role, I provided economic and statistical analysis for the Court.

## **II. ASSIGNMENT AND SUMMARY OF OPINIONS**

4. Counsel for the Nomura defendants (collectively, “Nomura”) have asked me to evaluate the opinions of plaintiff’s expert John Kilpatrick, and in particular the opinions

expressed in the Expert Report of John A. Kilpatrick, Ph.D. Concerning Accuracy of Appraisals, dated May 15, 2014 (“Kilpatrick Accuracy Report”) and his Expert Report of October 6, 2014 (“Kilpatrick Supplemental Report”). Based on my analysis, as described more fully below, I have reached the following conclusions.

5. Dr. Kilpatrick provides no evidence of any overstatement in the appraised values for the loans underlying the residential mortgage backed securities that are at issue in this lawsuit. Dr. Kilpatrick makes at least two fundamental errors in reaching his conclusion that the appraised values are overstated. First, Dr. Kilpatrick uses an unreliable level of statistical significance that causes him to overstate the number of loans for which the appraised value is significantly greater than the estimated value from his automated valuation model, the Greenfield AVM (“Greenfield AVM”). Second, in estimating the precision of the Greenfield AVM, Dr. Kilpatrick discards over 70% of the available data, not because those data are unreliable, but because doing so makes his AVM appear more precise. By discarding those data without justification, Dr. Kilpatrick overstates the precision of the Greenfield AVM. When these two errors are corrected, but otherwise making no changes to the Greenfield AVM (which is otherwise flawed), Dr. Kilpatrick’s AVM provides no evidence of any overstatement in the appraised values for the Nomura loans.

6. In addition to not providing evidence of any overstatement in the appraised values for the Nomura loans, the Greenfield AVM contains errors that further discredit Dr. Kilpatrick’s results and undermine his conclusions. First, the Greenfield AVM uses an incorrect methodology to calculate predicted values. Second, the Greenfield AVM fails to take into account property characteristics that affect value, including basic characteristics such as number of bedrooms. These errors cause Dr. Kilpatrick to systematically understate the value of the

properties associated with the Nomura loans. As a result, Dr. Kilpatrick's analysis is methodologically flawed and his conclusions are unreliable.

7. Dr. Kilpatrick's recalculation of the loan-to-value ("LTV") ratios for the Nomura loans results in LTV ratios that are too high for at least two reasons. First, Dr. Kilpatrick uses his AVM values to recalculate the LTV ratios. Because these AVM values are too low and the AVM values appear in the denominator of the calculations, the recalculated LTV ratios are too high. Second, the methodology Dr. Kilpatrick uses to recalculate the LTV ratios creates a censoring bias that causes his LTV ratios to be further overstated. When these errors are corrected, but otherwise making no changes to the Greenfield AVM, the recalculated LTV ratios are on average the same as their originally represented values.

### **III. DR. KILPATRICK'S OPINIONS AND ANALYSIS**

8. Dr. Kilpatrick concludes that "[t]he original appraised values of 208 of the 672 sample Nomura properties . . . were significantly higher than their true, credible, appraised values," and that "the LTV ratios for these sample properties were significantly higher than represented in the Prospectus Supplements for the Securitizations."<sup>1</sup>

9. Dr. Kilpatrick reaches these conclusions by using the Greenfield AVM, an AVM that was apparently created for this litigation. There are three components to Dr. Kilpatrick's analysis. The first component consists of a Greenfield AVM validation exercise.<sup>2</sup> In this component, Dr. Kilpatrick draws a 10% random sample of sales transactions with known sale prices and uses the Greenfield AVM to estimate the value of those properties, assuming the sale prices for those properties are unknown. He then compares the estimated value from the

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<sup>1</sup> Expert Report of John A. Kilpatrick, Ph.D., Concerning Accuracy of Appraisals, dated May 15, 2014 (including all appendices and exhibits) ("Kilpatrick Accuracy Report"), p. 3.

<sup>2</sup> *Id.*, pp. 46-57.

Greenfield AVM. Based on this comparison, Dr. Kilpatrick concludes that the forecast standard deviation of the Greenfield AVM, which is a measure of accuracy and precision, is 0.151.<sup>3</sup>

10. In the second component of his analysis, Dr. Kilpatrick uses the Greenfield AVM to value the sample of properties underlying the Securitizations at issue, as drawn by plaintiff's expert Charles Cowan (the "Nomura Sample Loans").<sup>4</sup> He finds that 208 of the Nomura Sample Loans had appraised values that exceed the Greenfield AVM value by at least one forecast standard deviation, where the forecast standard deviation is the value derived from the validation exercise (0.151).<sup>5</sup> Based on those findings, he concludes that the actual appraised values of those 208 properties "were significantly higher than their true, credible, appraised values."<sup>6</sup>

11. In the third component of his analysis, Dr. Kilpatrick uses the Greenfield AVM values for the Nomura Sample Loans to recalculate the LTV, combined LTV ("CLTV") and mixed LTV ("MLTV") ratios for the Nomura Sample Loans at the time of origination.<sup>7</sup> Dr. Kilpatrick concludes that the "Nomura sample loans had substantially higher MLTV ratios than represented."<sup>8</sup>

#### **IV. THE GREENFIELD AVM PROVIDES NO EVIDENCE OF ANY OVERSTATEMENT IN THE APPRAISED VALUES FOR THE NOMURA LOANS.**

12. Dr. Kilpatrick erroneously determines that the appraised values of the Nomura Sample Loans were significantly higher than their "true" value, as measured by the Greenfield AVM. Dr. Kilpatrick errs in at least two critical respects. First, he uses an unreliable level of statistical significance. Second, he overstates the precision of the Greenfield AVM. As I

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<sup>3</sup> *Id.*, p. 51. The forecast standard deviation is calculated as the square root of the average squared prediction error. *Id.*, Appendix 5-1, pp. 14-15.

<sup>4</sup> *Id.*, pp. 58-65.

<sup>5</sup> *Id.*, p. 62 n.161. In mathematical terms, Dr. Kilpatrick compares the appraised value to 1.151 multiplied by the Greenfield AVM value.

<sup>6</sup> *Id.*, p. 3.

<sup>7</sup> *Id.*, pp. 67-74.

<sup>8</sup> *Id.*, p. 69.

demonstrate below, when these errors are corrected, but otherwise making no changes to Dr. Kilpatrick's AVM, the results provide no evidence of any overstatement in the appraised values for the Nomura loans.

**A. Dr. Kilpatrick Uses an Unreliable Level of Statistical Significance.**

13. To determine whether an effect is statistically significant, statisticians formulate and test what is called a "null hypothesis."<sup>9</sup> In the context of Dr. Kilpatrick's analysis, the null hypothesis is that the appraised value is equal to the Greenfield AVM value. To test whether this null hypothesis should be rejected, statisticians calculate what is called a "*p*-value," which is the probability of observing the actual data (here, the difference between the appraised value and the Greenfield AVM value) if the null hypothesis is true.<sup>10</sup> If the *p*-value falls below a threshold called the significance level, the null hypothesis is rejected.<sup>11</sup> In the context of Dr. Kilpatrick's analysis, we would say the null hypothesis is rejected if the difference between the appraised value and the Greenfield AVM value is statistically significant.

14. The standard significance level in statistics and economics is 5%. A test with a significance level of 5% can be implemented by constructing what is called a "95% confidence interval." If the 95% confidence interval is constructed from repeated samples, the 95% confidence interval will contain the true value 95% of the time.<sup>12</sup> A 95% confidence interval is the interval within 1.96 forecast standard deviations of the value obtained from the sample.<sup>13</sup> By contrast, a 68% confidence interval is the interval within 1.0 forecast standard deviation of the sample value.<sup>14</sup>

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<sup>9</sup> See, e.g., D. Kaye and D. Freedman, "Reference Guide on Statistics," in *Reference Manual on Scientific Evidence*, 3<sup>rd</sup> ed., 2011, p. 241.

<sup>10</sup> *Id.*, p. 250.

<sup>11</sup> *Id.*, p. 251.

<sup>12</sup> *Id.*, p. 247.

<sup>13</sup> *Id.*, p. 244, n.89.

<sup>14</sup> *Id.*, p. 244.

15. As I have explained above, Dr. Kilpatrick's claim that the appraised value is significantly higher than the "true" value is based on comparing the appraised value to the value that is one forecast standard deviation above the Greenfield AVM value. Because Dr. Kilpatrick uses a factor of 1.0 forecast standard deviation instead of 1.96 forecast standard deviations, Dr. Kilpatrick's confidence interval is not a 95% confidence interval, but rather a 68% confidence interval.<sup>15</sup> Thus, the significance level used by Dr. Kilpatrick, which is equal to 100% minus the confidence level, is 32%. I have never encountered the use of a significance level as unreliable as 32% in reputable academic research. In my experience, the fields of econometrics and statistics universally require more rigor than Dr. Kilpatrick applies in using a 32% significance level. The use of a 32% significance level completely undermines the reliability and accuracy of Dr. Kilpatrick's opinions.

16. One way to demonstrate the unreliability of using a 32% significance level is to note that the significance level is the probability of incorrectly rejecting the null hypothesis (in this case, that there is no deviation between the Greenfield AVM and the appraised value), which is referred to as a "Type I error."<sup>16</sup> Thus, for each of the 208 properties that Dr. Kilpatrick claims had appraised values that "were significantly higher than their true, credible, appraised values,"<sup>17</sup> using Dr. Kilpatrick's significance level, there is almost a one-in-three chance that there is no difference between the appraised value and the Greenfield AVM's "true" value, and that a Type I error has been made by Dr. Kilpatrick instead. Thus, on average, Dr. Kilpatrick's

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<sup>15</sup> See *Id.*

<sup>16</sup> *Id.*, p. 251 n.100.

<sup>17</sup> Kilpatrick Accuracy Report, p. 3.

conclusion is wrong for 67 of the 208 properties (*i.e.*, 32% of the properties that Dr. Kilpatrick claims to have inflated appraisals).<sup>18</sup>

17. Even without taking into account the various problems with the Greenfield AVM, if the standard 5% significance level is used instead of Dr. Kilpatrick's 32% significance level, the number of loans for which the original appraisal value is significantly different from the Greenfield AVM value decreases from 208 (the number of loans with appraisals greater than 1.0 forecast standard deviation above the Greenfield AVM value) to 95 (the number of loans with appraisals greater than 1.96 forecast standard deviations above the Greenfield AVM value), a decrease of approximately 54%.<sup>19</sup> In other words, by using a level of statistical significance that is not scientifically accepted, Dr. Kilpatrick more than doubles the number of sampled appraisals that he finds inaccurate. This is a completely unreliable method of analysis.

18. Dr. Kilpatrick testified that such statistical standards do not apply to his model because his conclusions do not rely on "hypothesis testing" (*i.e.*, the procedure described in ¶ 12 above), but rather on establishing his AVM as a "baseline benchmark against which to measure whether the . . . appraisals are substantially false or not."<sup>20</sup> Dr. Kilpatrick provides no support for why he rejects hypothesis testing, which is the standard approach in both statistics and econometrics, in favor of the unsupported and unreliable approach he created for this litigation.<sup>21</sup>

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<sup>18</sup> This calculation only measures the number of properties for which Dr. Kilpatrick's conclusion is wrong due to his use of an unreliable significance level, and does not take into account Dr. Kilpatrick's other errors, which are discussed below.

<sup>19</sup> Given the unreliability of the Greenfield AVM as discussed below, the fact that some loans have appraisals that are greater than the Greenfield AVM value at the 5% significance level according to this calculation does not indicate that those appraisals are inflated.

<sup>20</sup> Deposition of John Kilpatrick, dated February 12, 2014 ("Kilpatrick Feb. 12 Dep."), at 247:19-248:4; *see also* Deposition of John Kilpatrick, dated June 30, 2014 ("Kilpatrick June 30 Dep."), at 50:18-19 ("And I am not using confidence in a hypothesis testing vein."); 130:19-25 ("I am not testing a hypothesis here. I am not engaged in hypothesis testing."); 372:11-21 ("I am not testing for whether this is the value or not. I am simply establishing a benchmark against which the original appraisals can be tested.").

<sup>21</sup> For the use of hypothesis testing in statistics, see, *e.g.*, D. Kaye and D. Freedman, "Reference Guide on Statistics," in *Reference Manual on Scientific Evidence*, 3rd ed., 2011, pp. 249-53. For the use of hypothesis testing



Dr. Kilpatrick attempted to justify his rejection of hypothesis testing by drawing an analogy to weighing an object:

Well, if I want to weigh something, I am not testing the hypothesis that the weight is two pounds. I am just putting it on the scale and see what the scale says. So I am not questioning what the weight of a two pound object is, and then conducting a series of empirical tests to try to deduce what that weight is. I simply put it on a scale. So I am not trying to create any hypothesis that needs to be tested about these original appraisals. I am simply establishing a benchmark and measuring them against that benchmark.<sup>22</sup>

19. Dr. Kilpatrick fails to recognize that the Greenfield AVM (like all AVMs) produces values that are subject to error. If you have three different objects, each weighing two pounds, a scale will report a weight of two pounds for each of the objects. In contrast, AVM value estimates are subject to error, which means that if you have three different properties, each with a market value of \$200,000, an AVM may estimate the value of those properties as \$180,000, \$200,000, and \$220,000, respectively.<sup>23</sup> Thus, as a matter of statistics and econometrics, in order to conclude that the difference between a Greenfield AVM value and an appraisal value is due to a difference between the appraisal value and the market value (as opposed to random error in the Greenfield AVM value), Dr. Kilpatrick must apply a confidence interval to his results, and the confidence interval Dr. Kilpatrick uses is incorrect.

20. Dr. Kilpatrick later claimed that when he used the 95% confidence interval, as statistical rigor requires, he still finds that a significant number of appraisals were overvalued.<sup>24</sup> However, Dr. Kilpatrick's analysis is still fundamentally flawed. Dr. Kilpatrick calculates the 95% confidence interval for a given subject property by taking the Greenfield AVM estimate (in

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in econometrics, *see, e.g.*, D. Rubinfeld, "Reference Guide on Multiple Regression," in *Reference Manual on Scientific Evidence*, 3rd ed., 2011, pp. 319-20.

<sup>22</sup> Kilpatrick June 30 Dep. at 373:7-21.

<sup>23</sup> The error to which the Greenfield AVM is subject is reflected in the fact that even under Dr. Kilpatrick's validation exercise (which overstates the precision of the Greenfield AVM as explained below), over 25% of the valued transactions have Greenfield AVM value estimates that are more than 13% away from the actual sale price. Kilpatrick Accuracy Report, p. 51.

<sup>24</sup> Kilpatrick Supplemental Report, at 4.

logarithmic terms), adding or subtracting 1.96 times the standard error for the prediction, and exponentiating the result.<sup>25</sup> However, he calculated the standard error using his post-cross-validation regression.<sup>26</sup> As I explain below, the use of the cross-validation filter in estimating the standard error results in a downward-biased estimate.

21. Further, Dr. Kilpatrick does not actually use 95% confidence intervals—by definition, a 95% confidence interval would include the sales prices of the Nomura Sample Loans 95% of the time (given that the Greenfield AVM is designed to predict sales prices). However, the purported 95% confidence intervals only include the sales prices 78% of the time—meaning they are really 78% confidence intervals. A 78% confidence interval provides a significance level of 22 percent. As explained above, the standard for statistical analysis is a 5% significance level. The use of a 22% significance level, like the original 32% significance level Dr. Kilpatrick used, is unprecedented, to my knowledge, and does not pass statistical muster.

**B. Dr. Kilpatrick Overstates the Precision of the Greenfield AVM.**

22. Another key input into Dr. Kilpatrick's analysis is the forecast standard deviation. The forecast standard deviation determines the width of the confidence intervals for the Greenfield AVM values and thus the range of appraised values that are not significantly different from the Greenfield AVM value. If the forecast standard deviation is understated, that suggests that more appraised values differ significantly from Greenfield AVM values than is actually the case.

23. Dr. Kilpatrick's methodology for calculating the forecast standard deviation for the Greenfield AVM is flawed and results in substantially understated confidence intervals. Dr. Kilpatrick's calculation of the forecast standard deviation is flawed because he discards 70%

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<sup>25</sup> *Id.* at 3.

<sup>26</sup> See ¶¶ 49-50, below, for analysis of the cross-validation filter.

of his data when calculating the forecast standard deviation through the application of improper data filters. Dr. Kilpatrick discards these data not because he has determined that they are unreliable, but rather because these data show that the Greenfield AVM is less precise than he claims.

### **1. Filters Applied by Dr. Kilpatrick**

24. As explained above, Dr. Kilpatrick calculates the forecast standard deviation of the Greenfield AVM based on the results of his validation exercise, in which he uses 90% of available homes in each county in the U.S. to test the ability of the Greenfield AVM to predict actual sales prices for the remaining 10% of the available homes in that county. Specifically, for the transactions in Dr. Kilpatrick's validation sample, Dr. Kilpatrick calculates the forecast standard deviation of the forecast error, which Dr. Kilpatrick defines as sale price minus the Greenfield AVM value, divided by the sale price.<sup>27</sup>

25. Dr. Kilpatrick, however, does not use the full validation sample to calculate the forecast standard deviation. Instead, Dr. Kilpatrick applies three data filters to the 10% holdout set before calculating the forecast standard deviation. First, Dr. Kilpatrick eliminates properties that have been built (or rebuilt) after 2008 because, according to him, "these transactions likely represent land or teardown sales."<sup>28</sup>

26. Second, Dr. Kilpatrick eliminates properties where the difference between the appraisal value and the Greenfield AVM value is greater than 100% because, according to him, "these are indicative of data errors or incorrect matches between the tax and deed data."<sup>29</sup> To implement this filter, Dr. Kilpatrick excludes properties unless the forecast error is less than 1 in

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<sup>27</sup> See the definition of "Rec\_Err" in Section 3.1 of Dr. Kilpatrick's Prepare\_CVTestData.R file.

<sup>28</sup> Kilpatrick Accuracy Report, p. 50. In the text of his report, Dr. Kilpatrick claims that he eliminated properties built (or rebuilt) after the end of 2007, but Dr. Kilpatrick's code indicates that he eliminated properties built (or rebuilt) after the end of 2008. See Section 2.2 of Dr. Kilpatrick's Prepare\_CVTestData.R file.

<sup>29</sup> Kilpatrick Accuracy Report, p. 50.

absolute value.<sup>30</sup> Given Dr. Kilpatrick's definition of the forecast error and the fact that the Greenfield AVM value cannot be less than zero, this filter is equivalent to excluding properties where the Greenfield AVM value is more than twice the sale price.

27. Third, Dr. Kilpatrick eliminates properties outside of the middle 30th percentile of sale price to assessed value ratio, purportedly "to eliminate suspect data."<sup>31</sup>

28. It is a basic principle of statistics that data should be excluded from an analysis only if those data are in some sense unreliable, and that unreliability can be determined only by investigating the data themselves, not just by observing whether the data are "high" or "low." As one textbook explains:

[T]he decision to exclude a data point from statistical analysis is equivalent to saying that it is "bad" data. However, just being unusual (extra-large or extra small, for instance), doesn't necessarily equate to being "bad" (in the sense of wrong, incorrect, biased, distorted, etc.). This means that the rationale for discarding a data point ought to consist of more than just the simple fact that the score is an outlier. Rather, it ought to focus first on the "truth" of the data point; that is, an argument that the observation is flawed in some identifiable way.<sup>32</sup>

29. Dr. Kilpatrick's application of the filters violates this basic principle of statistics. Although Dr. Kilpatrick claims that the data eliminated by these filters represent "data errors" or "suspect data,"<sup>33</sup> Dr. Kilpatrick provides no evidence that the data eliminated by the filters actually represent "data errors" or "suspect data." Instead, Dr. Kilpatrick's filters appear to be calculated only to increase the claimed accuracy of the Greenfield AVM. Indeed, Dr. Kilpatrick agreed that his filters were not intended to eliminate only unreliable data, testifying that "just because we eliminate something doesn't necessarily mean it's unreliable . . . Might there be something that we filtered out that was also reliable? Quite possibly so. But the – the filters

<sup>30</sup> See Section 3.2 of Dr. Kilpatrick's Prepare\_CVTestData.R file.

<sup>31</sup> Kilpatrick Accuracy Report, p. 47; Section 5.3 of Dr. Kilpatrick's Prepare\_CVTestData.R file.

<sup>32</sup> D. Orr, *Fundamentals of Applied Statistics and Surveys*, 1995, p. 142.

<sup>33</sup> Kilpatrick Accuracy Report, pp. 47 and 50.

don't exist to find unreliable. They exist to find reliable. And, so, there may be stuff that we're not using that – that's perfectly reliable but, nonetheless to – to cut this thing on the safe side, we're applying the filters.”<sup>34</sup> Contrary to Dr. Kilpatrick's claim, his approach does not “cut this thing on the safe side.” Instead, by excluding “perfectly reliable” data (which by definition includes property values that are very different from those predicted by the Greenfield AVM), he overstates the precision of the Greenfield AVM.

30. Indeed, Dr. Kilpatrick testified that he agreed the “purpose” of the data filters was to “achieve an optimum FSD.”<sup>35</sup> The “FSD” in this context is the forecast standard deviation, which measures the amount of variation between the predicted value and the actual value. Dr. Kilpatrick's approach is inconsistent with sound statistical practice. Instead of excluding data because it was faulty or non-representative, Dr. Kilpatrick excluded data because it did not fit his model in order to make his model appear more precise.

31. In the following sections, I discuss the two filters that have the greatest effect on the calculated forecast standard deviation of the Greenfield AVM: the “middle-30th-percentile” filter and the “high-forecast-error” filter.

**a) Middle-30th-Percentile Filter**

32. In testing the accuracy of his AVM, Dr. Kilpatrick eliminates properties where the ratio of sale price to assessed value is outside the middle 30th percentile for the respective county.<sup>36</sup> Dr. Kilpatrick excluded those properties purportedly to ensure the property sales used for validation are “indicative of a market transaction.”<sup>37</sup> He provides no support, however, for

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<sup>34</sup> Kilpatrick Feb. 12 Dep. at 345:19-346:11; *see also* Kilpatrick June 30 Dep., at 200-01 (“I am not saying that there might not be some good sales outside of [the middle 30 percentile]. I am saying to err on the side of caution, I am using the ones that are in the middle that are closest to the mean there.”); 309-310 (“We may very well have thrown out some good properties with the filtering of some bad transactions.”).

<sup>35</sup> Deposition of John Kilpatrick, dated November 13, 2014 (“Kilpatrick Nov. 13 Dep.”), at 156:24-157:9.

<sup>36</sup> Kilpatrick Accuracy Report, p. 47.

<sup>37</sup> *Id.*, p. 47 n.134.

the premise that 70% of all sales (*i.e.*, those outside the middle 30th percentile) were not executed at arm's length.

33. In considering the effects of such a filter, it is important to note that assessed value is one of the most important variables in the Greenfield AVM. The Greenfield AVM is based on a regression model in which the left-hand-side (or dependent) variable is the natural logarithm of the sale price and the right-hand-side (or explanatory) variables includes variables that purportedly affect the sale price.<sup>38</sup> Other than time data, the only right-hand-side variable included in all of the regressions used in the Greenfield AVM is assessed value.<sup>39</sup> Because the Greenfield AVM uses assessed value to explain sale price, and uses other properties in the county to value the target property, the Greenfield AVM will not perform well if the sale price to assessed value ratio of the target property is not similar to the sale price to assessed value ratio of the other properties in the county. Thus, by excluding properties with relatively high and low ratios from his validation exercise, Dr. Kilpatrick necessarily overstates the precision of the Greenfield AVM.

34. The overstatement in precision caused by Dr. Kilpatrick's use of the middle-30<sup>th</sup>-percentile filter can be demonstrated based on the properties of truncated distributions.<sup>40</sup> Focusing on the sale price and assessed value variables, and assuming for illustrative purposes that the coefficient of the assessed value variable is equal to one, the econometric model that is the basis of the Greenfield AVM can be written

$$\ln(\text{Sale Price}) = \ln(\text{Assessed Value}) + u$$

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<sup>38</sup> *Id.*, pp. 39-41.

<sup>39</sup> *Id.*

<sup>40</sup> I have previously published papers on the econometrics of truncated distributions. *See, e.g.*, J. Hausman and D. Wise, "Social Experimentation, Truncated Distributions, and Efficient Estimation," *Econometrica* 45, 1977, pp. 919-38; J. Hausman and D. Wise, "The Evaluation of Results from Truncated Samples," *Annals of Economic and Social Measurement* 5, 1976, pp. 421-45.

where “ln()” is the natural log function and “u” is the error term in the econometric model.<sup>41</sup>

Based on the properties of logarithms, the model can be rewritten as  $\ln(\text{Sale Price}/\text{Assessed Value}) = u$ . Thus, the error term in the model (which determines the precision of the AVM) is related to the sale price to assessed value ratio that Dr. Kilpatrick uses to apply his middle-30th-percentile filter. As a theoretical example, suppose that the error term has a normal distribution with zero mean and a variance (and standard deviation) of 1.0.<sup>42</sup> Applying Dr. Kilpatrick’s middle-30<sup>th</sup>-percentile filter truncates this distribution symmetrically. The variance of a symmetrically truncated (at  $-a$  and  $a$ ) standard normal distribution is

$$\text{Var}(u | -a < u < a) = 1 \frac{2a\phi(a)}{\Phi(a) - \Phi(-a)}$$

where  $\phi()$  and  $\Phi()$  are the probability density function and cumulative distribution function of the standard normal distribution, respectively.<sup>43</sup> The value of  $a$  that corresponds to a middle-30<sup>th</sup>-percentile filter is 0.385.<sup>44</sup> Applying the formula, the variance of the error term after the middle-30th-percentile filter has been applied is 0.0485, which corresponds to a standard deviation of 0.22. Thus, in this example, Dr. Kilpatrick’s application of the middle-30th-percentile filter creates a severe downward bias in the standard deviation, from 1 (the standard deviation without truncation) to 0.22. This calculation demonstrates that applying the middle-30<sup>th</sup>-percentile filter creates a significant downward bias in calculating a forecast standard deviation. Below, I quantify the bias within the context of the Greenfield AVM model.

<sup>41</sup> As noted above, the only other right-hand-side variables that appear in all regressions for the Greenfield AVM consist of a quadratic time trend.

<sup>42</sup> The assumption that the standard deviation is one is a normalization and does not affect the calculation, which is done in percentage terms.

<sup>43</sup> See N. Johnson, S. Kotz, and N. Balakrishnan, *Continuous Univariate Distributions*, Volume 1, 2<sup>nd</sup> ed., 1994, p. 158.

<sup>44</sup> Note that  $\Phi(0.385) = 0.65$  and  $\Phi(-0.385) = 0.35$ , where  $\Phi()$  is the cumulative distribution function of the standard normal distribution.

35. In his discussion of the middle-30<sup>th</sup>-percentile filter, Dr. Kilpatrick claims that “on average, half of assessed value ratios in a county should be within 15% either way of the median (30% total),” and that he “selected this middle 30% of the sales price to assessed value ratio (‘SPAVR’) to ensure that these sales are indeed indicative of a market transaction and therefore should be used to validate the Greenfield AVM.”<sup>45</sup> Dr. Kilpatrick fails to note that, while he tries to explain his filter by claiming that half of all transactions *should* fall within the middle 30th percentile, that is not the case; only 30% of transactions have assessed value ratios within 15% of the median, so that his filter actually excludes close to 70% of transactions.<sup>46</sup> Defense Exhibit 2805 (“DX-2805”) shows visually just how much of the data Dr. Kilpatrick excludes. Dr. Kilpatrick provides no evidence that any of the excluded transactions are not “indicative of a market transaction,” much less that close to 70% of all transactions should be excluded. As explained above, such evidence is required for Dr. Kilpatrick’s filter to be justified, because the “decision to exclude a data point from statistical analysis is equivalent to saying that it is ‘bad’ data.”<sup>47</sup>

#### **b) High-Forecast-Error Filter**

36. In evaluating the accuracy of his AVM, Dr. Kilpatrick also excludes properties where the Greenfield AVM value is more than twice the sale price. By definition, these properties have high forecast errors, so by excluding these properties Dr. Kilpatrick again overstates the precision of the Greenfield AVM. As noted above, although Dr. Kilpatrick eliminates these properties because he believes the results “are indicative of data errors or incorrect matches between the tax and deed data,”<sup>48</sup> he performs no analysis of any such errors

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<sup>45</sup> Kilpatrick Accuracy Report, p. 47 n.134.

<sup>46</sup> If the middle-30<sup>th</sup>-percentile filter is not applied, the number of properties increases from 675,613 to 2,169,144.

<sup>47</sup> D. Orr, *Fundamentals of Applied Statistics and Surveys*, 1995, p. 142.

<sup>48</sup> Kilpatrick Accuracy Report, p. 50.



or incorrect matches for these properties. Dr. Kilpatrick testified that he did not do anything to test whether properties with high forecast error actually represented spurious data, noting that it “would have been extremely time consuming.”<sup>49</sup>

## **2. Quantifying the Effect of Dr. Kilpatrick’s Filters**

### **a) Quantifying the Effect by Removing Dr. Kilpatrick’s Filters**

37. In my opinion, neither filter described above is defensible under established principles of econometrics and statistics. In Defense Exhibit 2806 (“DX-2806”), attached to this affidavit, I quantify the amount by which Dr. Kilpatrick has overstated the precision of the Greenfield AVM by applying those inappropriate filters. In the first column of DX-2806, I calculate the forecast standard deviation in Dr. Kilpatrick’s validation sample if the middle-30th-percentile filter is not applied (but the high-forecast-error filter is applied), then determine the number of Nomura Sample Loans with an appraised value significantly greater than the Greenfield AVM value, using the 5% significance level. In the second column, I perform the same calculations when the high-forecast-error filter is not applied (but the middle-30<sup>th</sup>-percentile filter is applied). In the third column, I perform the same calculations when neither filter is applied.

38. As DX-2806 shows, when the middle-30<sup>th</sup>-percentile filter is not applied, the forecast standard deviation increases to 0.238, and the number of Nomura loans with an appraised value significantly greater than the Greenfield AVM value decreases from 95 to 43, or 6.40% of the 672 Nomura Sample Loans valued by the Greenfield AVM. If the high-forecast-error filter is not applied (but the middle-30<sup>th</sup>-percentile filter is applied), the forecast standard deviation increases to 0.438, and the number of Nomura Sample Loans with an appraised value significantly greater than the Greenfield AVM value decreases even further to 4, or 0.6% of the

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<sup>49</sup> Kilpatrick June 30 Dep. at 255:6-15.

672 Nomura Sample Loans valued by the Greenfield AVM. If neither filter is applied, the forecast standard deviation explodes to over 2,000, and, under Dr. Kilpatrick's methodology, none of the Nomura loans would be considered to have an appraised value significantly greater than the Greenfield AVM value.

39. The results in DX-2806 demonstrate that Dr. Kilpatrick's analysis provides no scientifically valid evidence of any overstatement in the appraised values for the Nomura loans. To see why this is the case, note that if the null hypothesis is correct (*i.e.*, there is no difference between the appraised values and the Greenfield AVM values) then, when testing the null hypothesis at the 5% significance level, on average we would expect approximately 2.5% of the loans to have appraised values significantly greater than the Greenfield AVM value. Thus, a finding that 2.5% or less of the Nomura loans have appraised values that are significantly greater than the Greenfield AVM value is consistent with the null hypothesis. As Exhibit 4 shows, the percentage of Nomura loans that have appraised values significantly greater than the Greenfield AVM value is only six tenths of 1% when the high-forecast-error filter is not applied, and 0% when neither the middle-30<sup>th</sup>-percentile filter nor the high-forecast-error filter is applied. Thus, both of these findings are consistent with the null hypothesis—namely, that there is no difference between the appraised values and the Greenfield AVM values.

40. The finding that 6.40% of the Nomura loans have appraised values that are significantly greater than the Greenfield AVM value when the middle-30<sup>th</sup>-percentile filter is not applied initially appears to be inconsistent with the null hypothesis, because it is greater than the expected value of 2.5%. However, the expected value of 2.5% is based on the distribution of errors following a normal distribution. As Dr. Kilpatrick acknowledges, the distribution of errors from the Greenfield AVM are non-normal. In particular, when all of the filters are applied, the

errors from the Greenfield AVM are leptokurtotic—a distribution that has “fat tails” compared to a normal distribution, in the sense that the probability of extreme outcomes is more likely.<sup>50</sup>

41. To account for the non-normality of errors, in the last three rows of Exhibit 4, I perform the same analysis on the validation sample as I did on the Nomura Sample Loans. In particular, I calculate the forecast standard deviation in Dr. Kilpatrick’s validation sample if the middle-30<sup>th</sup>-percentile filter is not applied, and then determine the number of loans in the validation sample with a sale price significantly greater than the Greenfield AVM value, using the 5% level of significance. Because the null hypothesis is assumed to hold for the validation sample (*i.e.*, there is no difference between the sale price and the Greenfield AVM values), the results of this analysis indicate the percentage of loans that can be expected to have values significantly greater than the Greenfield AVM value under the null hypothesis.

42. As DX-2806 shows, when the middle-30<sup>th</sup>-percentile filter is not applied, 5.37% of the properties in the validation sample have sale prices significantly greater than the Greenfield AVM value. This percentage is greater than the 2.5% that would occur if the Greenfield AVM errors followed a normal distribution, and indicates that the errors from the Greenfield AVM have a leptokurtotic distribution with “fat tails.” A leptokurtotic distribution is a distribution that has “fat tails” compared to a normal distribution, in the sense that the probability of extreme outcomes is more likely.<sup>51</sup> Therefore, if the percentage of loans in the Nomura sample with appraised values significantly greater than the Greenfield AVM value is similar to the percentage of validation sample loans with sale prices significantly greater than the Greenfield AVM value, that finding indicates that, as with the validation sample, there is no significant difference between Greenfield AVM values and appraised values. In fact, the

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<sup>50</sup> Kilpatrick Accuracy Report, p. 53.

<sup>51</sup> See, *e.g.*, J. Stock and M. Watson, *Introduction to Econometrics*, 2nd ed., 2007, pp. 26-8.

percentage of properties in the validation sample with sale prices significantly greater than the Greenfield AVM value (5.37%) is close to the percentage of loans in the Nomura sample with appraised values significantly greater than the Greenfield AVM value (6.40%). Thus, I conclude that the results when the middle-30th-percentile filter is not applied are consistent with the null hypothesis that the appraised values are equal to the Greenfield AVM value. In other words, the Greenfield AVM provides no evidence of any overstatement in the appraised values.

**b) Quantifying the Effect by Applying the Middle-30<sup>th</sup>- Percentile Filter to the Nomura Loans**

43. The forecast standard deviation Dr. Kilpatrick uses in his analysis is based on a sample of properties in which properties with a ratio of sale price to assessed value that is outside of the middle 30<sup>th</sup> percentile have been excluded. Because limiting the sample in this manner reduces the forecast standard deviation, as I have demonstrated above, it is appropriate to apply this same forecast standard deviation only to loans that also fall within the middle 30<sup>th</sup> percentile. Thus, Dr. Kilpatrick errs when he applies a forecast standard deviation calculated from a sample where the middle-30th-percentile filter has been applied (*i.e.*, the validation sample he uses to calculate a forecast standard deviation of 0.151) to a sample where the middle-30th-percentile filter has not been applied (*i.e.*, the Nomura Sample Loans). If the middle-30th-percentile filter is also applied to the Nomura Sample Loans, Dr. Kilpatrick's analysis leads to the conclusion that the Greenfield AVM provides no evidence of any overstatement in the appraised values.

44. Among the 672 Nomura loans in Dr. Kilpatrick's analysis, 305 are part of purchase transactions for which sale prices are available and thus a sale price to assessed value ratio can be calculated. For 302 of these 305 loans, the sale price to assessed value ratio can be compared to the middle-30<sup>th</sup>-percentile cutoffs used by Dr. Kilpatrick. Out of these 302 loans, 220 (72.8%) have sale price to assessed value ratios that are outside of the cutoffs used by

Dr. Kilpatrick, and thus it is inappropriate to apply Dr. Kilpatrick's forecast standard deviation to these loans.

45. Out of the 82 Nomura Sample Loans remaining after the middle-30th-percentile filter is applied, 3 loans (3.66%) have appraisals that are significantly greater than the Greenfield AVM value using the standard 5% significance level. As I discuss above, under the null hypothesis that there is no difference between the appraised values and the Greenfield AVM values, when testing at the 5% significance level, we would expect approximately 2.5% of the loans to have appraised values significantly greater than the Greenfield AVM value, assuming the errors follow a normal distribution. However, as is discussed above, the errors from the Greenfield AVM are non-normal. To account for this non-normality, I calculate the percentage of loans in Dr. Kilpatrick's validation sample (after the middle-30th-percentile filter has been applied) that have sale prices significantly greater than the Greenfield AVM value at the 5% significance level. Because the null hypothesis is assumed to hold for the validation sample (*i.e.*, there is no difference between the sale price and the Greenfield AVM values), the results of this analysis indicate the percentage of loans that can be expected to have values significantly greater than the Greenfield AVM value under the null hypothesis. Thus, if the percentage of loans in the Nomura sample (after applying the middle-30th-percentile filter) with appraised values significantly greater than the Greenfield AVM value is similar to the percentage of loans in the validation sample (after applying the middle-30th-percentile filter) with sale prices significantly greater than the Greenfield AVM value, that finding indicates that, as with the validation sample, there is no significant difference between Greenfield AVM values and appraised values.

46. The percentage of loans in the validation sample inside the middle 30th percentile with sale prices significantly greater than the Greenfield AVM value is 4.39%. The percentage

of Nomura Sample Loans inside the middle 30th percentile with appraised values significantly greater than the Greenfield AVM value is 3.66%, which is similar to (and in fact less than) the 4.39% figure derived from the validation sample. Thus, I conclude that the results are consistent with the null hypothesis that the appraised values are equal to the Greenfield AVM values. In other words, again, the Greenfield AVM provides no evidence of any overstatement in the appraised values.

**3. Dr. Kilpatrick's Inconsistent Application of Filters Means That Many Transactions That Are Excluded from Some of His Analyses Are Included in Other of His Analyses**

47. Dr. Kilpatrick applies the middle-30th-percentile and high-forecast-error filters to the transactions included in the 10% holdout sample for his validation exercise, but does not apply those filters when valuing the Nomura Sample Loans. As I demonstrate in this section, Dr. Kilpatrick's inconsistent application of filters means that many transactions that are excluded from some of his analyses are included in other of his analyses. This inconsistency further suggests that Dr. Kilpatrick applied his filters in order to "optimize" his results, not to exclude unreliable data.

**a) Comparison of the Transactions Filtered Out by the Middle-30<sup>th</sup>-Percentile Filter and the Transactions Used to Value the Nomura Sample Loans**

48. In valuing the Nomura Sample Loans, Dr. Kilpatrick used 892,984 unique transactions as comparables. However, 56,533 of those transactions were originally part of Dr. Kilpatrick's 10% holdout sample in the validation exercise but were removed by Dr. Kilpatrick's application of the middle-30<sup>th</sup>-percentile filter. Thus, Dr. Kilpatrick considers those 56,533 transactions reliable for the purpose of valuing the Nomura Sample Loans, but unreliable for the purpose of calculating the forecast standard deviation.

**b) Comparison of the Transactions Filtered Out by the Cross-Validation Filter and the Transactions Used in the Validation Exercise**

49. In addition to the middle-30th-percentile and high-forecast-error filters discussed above, Dr. Kilpatrick applies a cross-validation filter when valuing both the properties in the 10% holdout sample and the Nomura Sample Loans. After estimating the preliminary regression for a subject property, Dr. Kilpatrick calculates the cross-validation error for each transaction included in the regression. Dr. Kilpatrick then excludes transactions with cross-validation errors that exceed a certain threshold from the final regression used to value the property.<sup>52</sup>

50. In Dr. Kilpatrick's valuation of the Nomura Sample Loans, 193,788 transactions were always removed by his cross-validation filter after being included in a preliminary regression. However, 5,107 of those same transactions appear in Dr. Kilpatrick's 10% holdout sample and are used to calculate the forecast standard deviation in Dr. Kilpatrick's validation exercise. Thus, Dr. Kilpatrick considers those 5,107 transactions reliable for the purpose of calculating the forecast standard deviation, but unreliable for the purpose of valuing the Nomura Sample Loans.

**C. Conclusion**

51. My analysis in this section has demonstrated that by using an incorrect significance level and overstating the precision of the Greenfield AVM, Dr. Kilpatrick erroneously concludes that the Nomura Sample Loans have significantly overstated values. When these errors are corrected, but otherwise making no changes to the Greenfield AVM, the results provide no evidence of any overstatement in the appraised values for the Nomura loans.

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<sup>52</sup> Kilpatrick Accuracy Report, pp. 34-35. Dr. Kilpatrick uses a cross-validation threshold of 0.25. *Id.*, pp. 59-61

**V. ERRORS IN THE GREENFIELD AVM FURTHER DISCREDIT DR. KILPATRICK'S CLAIMS.**

52. In the preceding section, I demonstrated that even if no changes are made to the Greenfield AVM, the Greenfield AVM provides no evidence of any overstatement in the appraised values for the Nomura loans. As I now discuss, there are errors embedded in the Greenfield AVM that cause it to understate the value of the properties Dr. Kilpatrick attempts to value. These errors further discredit Dr. Kilpatrick's claims and undermine his conclusions.

**A. Dr. Kilpatrick's Prediction Error**

53. Dr. Kilpatrick's prediction error arises from the fact that the regression model in the Greenfield AVM predicts the natural logarithm of the sale price, not the sale price itself, and thus a transformation is required in order to generate a predicted value in dollar terms.<sup>53</sup> As I explain in the rest of this section, the method used by Dr. Kilpatrick to transform the natural logarithm of the sale price into a predicted value is incorrect. When the correct method is used, the predicted values of the Nomura Sample Loans are substantially higher than those reported by Dr. Kilpatrick.

54. As discussed above, the regression model in the Greenfield AVM is a log-linear model in which the left-hand-side variable is the natural logarithm of the sale price. Because the goal of the AVM is to predict the sale price itself, and not the natural logarithm of the sale price, the predicted value of the left-hand-side variable must be transformed. Dr. Kilpatrick recognizes this fact, but he erroneously performs this transformation by exponentiating the predicted value of the left-hand-side variable.<sup>54</sup> Dr. Kilpatrick's approach is a basic error in econometrics that results in a predicted value that necessarily understates the true predicted value. As an

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<sup>53</sup> The natural logarithm of a number  $x$  is the power to which Euler's number ( $e \approx 2.71828$ ) would have to be raised to equal  $x$  (i.e.,  $e^{\ln(x)} = x$ ).

<sup>54</sup> Kilpatrick Accuracy Report, Appendix 5-1, p. 34 ("[P]redicted values are converted from their natural log form to full dollar values using the exp() function and saved as a predicted value").



undergraduate textbook in econometrics states, Dr. Kilpatrick's approach "does not work; in fact it will systematically *underestimate* the expected value."<sup>55</sup> Dr. Kilpatrick's error is that he fails to account for the variance of the error term in the regression model when generating the predicted value.<sup>56</sup>

55. To correct Dr. Kilpatrick's prediction error, it is necessary to take into account the standard error of the regressions in the Greenfield AVM.<sup>57</sup> In estimating the standard errors of the regressions, it is important to account for Dr. Kilpatrick's cross-validation filter, which I discuss above. Because the standard error of a regression depends on the residuals of the individual observations included in that regression, the standard errors of Dr. Kilpatrick's post-cross-validation regressions are less than the true standard errors of those regressions. Thus, Dr. Kilpatrick's estimated standard errors are downward biased and unreliable. To estimate the true standard errors of the regressions, I calculate the standard error for a given regression by applying the coefficients estimated by Dr. Kilpatrick for that regression to the entire pre-cross-validation sample for that regression. To correct Dr. Kilpatrick's prediction error for a given property, I correct both Dr. Kilpatrick's prediction from the OLS model using the estimated standard error of the OLS regression for that property and Dr. Kilpatrick's prediction from the OLSXY model using the estimated standard error of the OLSXY regression for that property. I then take the average of the corrected OLS predicted value and the corrected OLSXY predicted value. When Dr. Kilpatrick's prediction error is removed, the Greenfield AVM predicted value for the Nomura Sample Loans increases by an average of approximately \$39,500 per property or,

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<sup>55</sup> J. Wooldridge, *Introductory Econometrics: A Modern Approach*, 3<sup>rd</sup> ed., 2006, p. 219.

<sup>56</sup> In particular, Dr. Kilpatrick's predicted value is  $\exp(\ln(y))$ , but the correct prediction is  $\exp(\sigma^2/2)\exp(\ln(y))$ , where  $\sigma$  is the estimated standard error of the regression. See *id.*, p. 219. Because  $\exp(\sigma^2/2)$  is necessarily greater than one, the correction prediction is necessarily greater than Dr. Kilpatrick's prediction.

<sup>57</sup> There are two regression models (OLS and OLSXY) associated with each property that is valued.

in percentage terms, by approximately 18.0%.<sup>58</sup> The effect of the cross-validation filter can be seen in Defense Exhibit 2807 (“DX-2807”), attached to this affidavit.

56. Dr. Kilpatrick has claimed that my correction was “improper[]” because when calculating the standard error I did not take into account the trimming (cross-validation) he performed.<sup>59</sup> Dr. Kilpatrick is incorrect. Dr. Kilpatrick claims that his cross-validation filter removes “suspect transaction[s]” that are “statistical outlier[s] or non-representative data point[s],”<sup>60</sup> and that his cross-validation process “ensures that comparable sales used within the modeling process are legitimate market transactions.”<sup>61</sup> Dr. Kilpatrick, however, performs no analysis to determine whether the data he excludes via his cross-validation process are non-representative or otherwise problematic. Instead, much like the filters Dr. Kilpatrick applies in his validation exercise, Dr. Kilpatrick’s cross-validation filter appears designed to exclude data that do not fit his model in an attempt to make his model appear more precise. However, even if Dr. Kilpatrick is right and certain “suspect” data needs to be removed, the removal of his prediction error still eliminates the entirety of his opinions. When transactions with sales prices or tax assessed values less than \$10,000 or greater than \$3,000,000 are excluded (*i.e.*, properties which would actually be outliers, unlike the “outliers” Dr. Kilpatrick never examined), the average adjustment for the error Dr. Kilpatrick committed is 8.8%. That number is nearly identical to the 8.92% “average” bias that Dr. Kilpatrick finds for the Nomura Sample Loans.

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<sup>58</sup> It was not possible to replicate, using the code and data turned over by Dr. Kilpatrick, the predicted values reported by Dr. Kilpatrick for every property included in his analysis. I was able to replicate Dr. Kilpatrick’s predicted values for 671 out of 672 properties. My calculation is based on the 671 properties for which Dr. Kilpatrick’s code and data produced the reported values and for which the Greenfield AVM produced a prediction after making the changes I describe.

<sup>59</sup> Kilpatrick Nov. 13 Dep. at 328:8-329:13.

<sup>60</sup> Kilpatrick Accuracy Report, p. 33 n.84.

<sup>61</sup> *Id.*, p. 60 n.158.

**B. Omitted Variables Bias**

57. Another key shortcoming of the Greenfield AVM is that it fails to take into account numerous property characteristics that would be expected to affect value and can be taken into account by appraisers, such as the number of bedrooms, views, swimming pools, quality of schools, and access to public transportation. As I explain in the rest of this section, this failure means that the Greenfield AVM is subject to what is known in econometrics as “omitted variables bias.” Dr. Kilpatrick’s analysis provides no evidence that the omitted characteristics are properly excluded from the Greenfield AVM. To illustrate the effect of omitted variables bias on the Greenfield AVM, I recalculate the predicted values from the Greenfield AVM when one of the omitted variables (the number of bedrooms) is included in the analysis, and then I demonstrate that omitted variables bias causes Dr. Kilpatrick to systematically understate the value of the Nomura Sample Loans that Dr. Kilpatrick claims to have significantly overstated appraised values.

58. Omitted variables bias occurs where relevant explanatory variables are omitted from the regression model. In general, omitting relevant variables means that both the estimated coefficients and their standard errors are biased.<sup>62</sup>

59. The Greenfield AVM is subject to omitted variables bias because it includes very few “right-hand-side” (or independent) variables. As I discuss above, the only variables included in all of Dr. Kilpatrick’s regressions are assessed value and a quadratic time trend. In the “OLSXY” version of Dr. Kilpatrick’s regression model, variables based on latitude and longitude are also added to the model. If sufficient data are available, the Greenfield AVM

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<sup>62</sup> For an introduction to omitted variables bias, see, e.g., W. Greene, *Econometric Analysis*, 5<sup>th</sup> ed., 2003, pp. 148-9. The importance of omitted variables bias has been recognized in the econometrics literature since the 1950s. See, e.g., Z. Griliches, “Specification Bias in Estimates of Production Functions,” *Journal of Farm Economics* 39, 1957, pp. 8-20.

regression model also includes up to four additional right-hand-side variables: home size, year built, lot size, and number of bathrooms.<sup>63</sup> Thus, the Greenfield AVM omits numerous characteristics that would be expected to affect value (and can be taken into account by appraisers), such as number of bedrooms, views, swimming pools, quality of schools, access to public transportation, just to name a few.

60. Dr. Kilpatrick claims that he “determined that incorporating other variables . . . was not necessary because of the high statistical correlations with the variables [he] had already chosen.”<sup>64</sup> However, the only empirical analysis Dr. Kilpatrick provides in support of this statement is a correlation analysis between home size (which is an optional variable) and the number of bedrooms (which is never included in the model).<sup>65</sup> Dr. Kilpatrick finds that the correlation between these two variables is 0.55, which he characterizes as a “high correlation,”<sup>66</sup> and claims “indicates that for modeling value influence, living space is a proxy for number of bedrooms.”<sup>67</sup> Contrary to Dr. Kilpatrick’s claim, a correlation of 0.55 is not indicative of “high correlation.” Correlation coefficients range (in absolute value) from 0 (indicating no linear association between the variables) to 1 (indicating a perfect linear relationship between the variables), and thus a correlation of 0.55 is better characterized as moderate.<sup>68</sup>

61. Furthermore, Dr. Kilpatrick’s correlation analysis fails to demonstrate that it is proper to exclude the number of bedrooms (or any other variable) as a right-hand-side variable. As a matter of econometrics, omission of a variable may lead to substantial bias even if it has a

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<sup>63</sup> Kilpatrick Accuracy Report, pp. 39-40.

<sup>64</sup> *Id.*, p. 37.

<sup>65</sup> *Id.*, Appendix 6-4, p. 1.

<sup>66</sup> *Id.*, p. 36.

<sup>67</sup> *Id.*, p. 36 n.96.

<sup>68</sup> See, e.g., D. Kaye and D. Freedman, “Reference Guide on Statistics,” in *Reference Manual on Scientific Evidence*, 3rd ed., 2011, pp. 261-2 (correlation of 0.5 characterized as “moderate”).

“high” correlation with other variables that are included in the regression.<sup>69</sup> As I now demonstrate, by excluding the number of bedrooms as a right-hand-side variable, Dr. Kilpatrick systematically understates the value the Greenfield AVM generates for the Nomura Sample Loans that he claims have significantly overstated appraised values.

62. As Dr. Kilpatrick acknowledges, information on the number of bedrooms is available for homes in 42% of the counties in the CoreLogic data he used.<sup>70</sup> Thus, to determine directly the impact of excluding this variable, I reestimate the Greenfield AVM for the Nomura Sample Loans, making only two changes. First, I add the number of bedrooms to the list of required variables in the Greenfield AVM. Second, I correct Dr. Kilpatrick’s prediction error, as discussed in the previous section.

63. Making these changes, I find that for the properties that Dr. Kilpatrick claims have significantly overstated appraised values, treating the number of bedrooms as a required variable and correcting Dr. Kilpatrick’s prediction error increases the Greenfield AVM predicted value by an average of approximately \$24,000 per property, or an average of approximately 12.3% on a percentage basis.<sup>71</sup> Considering only the effect of treating the number of bedrooms as a required variable, the average increase in the Greenfield AVM predicted value is approximately \$2,300 per property, or approximately 0.6% on a percentage basis.

64. These results directly contradict Dr. Kilpatrick’s claim that the number of bedrooms variable is properly excluded from his model. To the contrary, I have demonstrated that, by excluding the number of bedrooms variable, Dr. Kilpatrick systematically understates

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<sup>69</sup> See, e.g., W. Greene, *Econometric Analysis*, 5<sup>th</sup> ed., 2003, p. 58.

<sup>70</sup> Kilpatrick Accuracy Report, p. 36 n.95.

<sup>71</sup> This calculation is based on 121 Nomura Sample Loans (1) which are among the 208 sample properties that Dr. Kilpatrick claims have original appraised values that are significantly higher than their true appraised values; (2) for which the Greenfield AVM generates predicted values after adjustment; and (3) for which the number of bedrooms in the CoreLogic Match and Append Data used by Dr. Kilpatrick is not missing and ranges between 1 and 9 (I exclude one property with a reported number of bedrooms equal to 0).

the value that the Greenfield AVM generates for the Nomura Sample Loans that he claims have significantly overstated appraisals.

65. Dr. Kilpatrick's analysis thus provides no evidence that variables, including but not limited to the number of bedrooms, are properly excluded from the Greenfield AVM. By failing to account for key variables affecting property values—like bedrooms, swimming pools, views, quality of schools and access to public transportation—even where data concerning such variables was readily available, Dr. Kilpatrick biased his model to undervalue properties. This error further undermines Dr. Kilpatrick's conclusions and the reliability of the Greenfield AVM.

#### **VI. DR. KILPATRICK DOES NOT SHOW THAT THE ORIGINALLY REPRESENTED LTV RATIOS WERE UNDERSTATED.**

66. Dr. Kilpatrick uses the Greenfield AVM results to recalculate the LTV ratios for the Nomura Sample Loans at the time of origination.<sup>72</sup> Dr. Kilpatrick's recalculated LTV ratio is equal to the principal balance of the loan at the time of origination divided by the lesser of the Greenfield AVM value, the original appraised value, or (if the loan is a purchase loan) the sale price of the mortgaged property.<sup>73</sup> There are at least two errors in Dr. Kilpatrick's recalculation of the LTV ratios that cause him to overstate the true LTV ratio. First, as I have shown in the previous section, errors in the Greenfield AVM cause the Greenfield AVM to undervalue the Nomura Sample Loans. Because the Greenfield AVM value appears in the denominator of the LTV ratio, a Greenfield AVM value that is too low causes the recalculated LTV ratio to be too high. Second, Dr. Kilpatrick's methodology for recalculating the LTV ratio (in particular, using the lesser of the Greenfield AVM value, the original appraised value, or the sale price in the denominator) is an example of what is known in statistics and econometrics as "censoring," and

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<sup>72</sup> Kilpatrick Accuracy Report, p. 67.

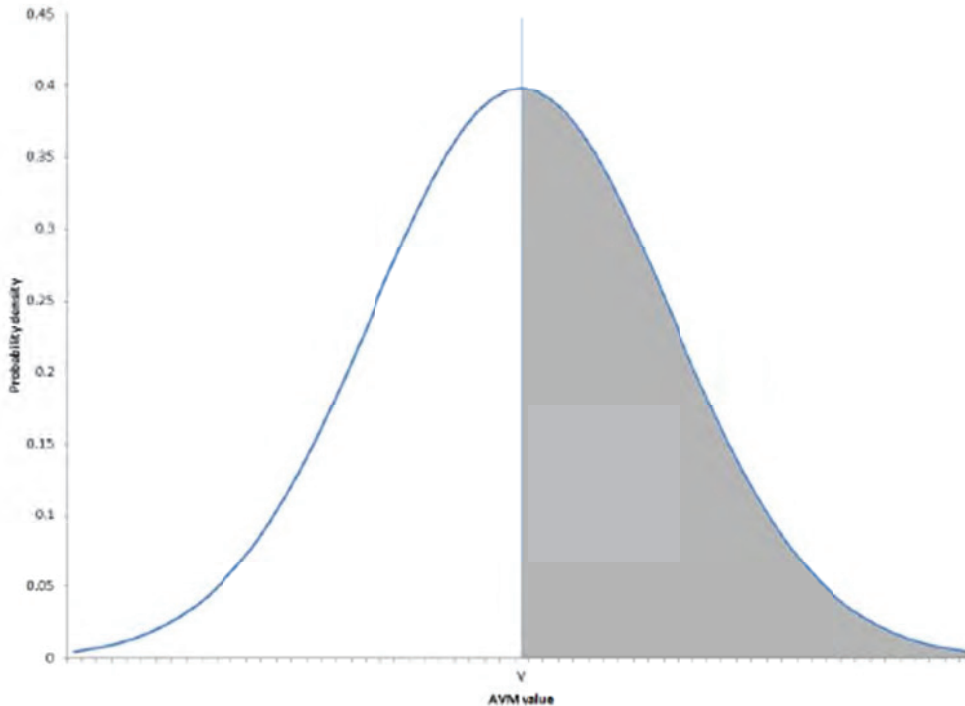
<sup>73</sup> *Id.*, p. 68. Dr. Kilpatrick contends that he uses the lesser of appraised value, sale price and Greenfield AVM value when calculating LTV because it comports with "common industry practice" and the offering materials in this case. However, Dr. Kilpatrick's selective reliance on the Greenfield AVM is inconsistent with his contention that the Greenfield AVM provides a more accurate value than appraisals and sales.

this censoring creates a bias that causes Dr. Kilpatrick to further overstate the recalculated LTV ratios. In the remainder of this section, I explain how Dr. Kilpatrick's methodology creates censoring bias and demonstrate that when Dr. Kilpatrick's errors are corrected, the recalculated LTV ratios are on average the same as their originally represented values.

67. Dr. Kilpatrick's methodology would cause him to overstate the recalculated LTV ratios even if on average the values originally used to calculate the LTV ratios were the same as the Greenfield AVM values. The bias created by Dr. Kilpatrick's methodology can be illustrated graphically by reference to Figure 1 below, which illustrates a censored distribution. The bell-shaped curve in Figure 1 represents the probability density of the Greenfield AVM value.<sup>74</sup> The distribution of the Greenfield AVM value is centered at what is labeled "V," the value originally used to calculate the LTV ratio (which is the lesser of the appraised value and any actual sale price), but it is also possible for the Greenfield AVM value to be above or below V.

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<sup>74</sup> Figure 1 uses a normal distribution for illustrative purposes, but the bias caused by censoring does not depend on the assumption of a normal distribution.

**Figure 1**

68. The shaded area in Figure 1 represents the part of the distribution that is censored by Dr. Kilpatrick's methodology. If the Greenfield AVM value is less than both the sale price of the home and the appraised value, then Dr. Kilpatrick uses the Greenfield AVM value in the denominator of the LTV ratio. If the Greenfield AVM value is greater than either the sale price or the appraised value (*i.e.*, in the shaded area), Dr. Kilpatrick instead uses the lesser of those two numbers in the denominator of the LTV ratio.

69. If Dr. Kilpatrick used only the Greenfield AVM value (and if the Greenfield AVM value were otherwise unbiased, which is incorrect for the reasons I have explained above), then using only the Greenfield AVM value (*i.e.*, not censoring the distribution) would on average be correct. Sometimes the AVM value would be above V (meaning there is a positive error in the AVM value), and sometimes the AVM value would be below V (meaning there is a negative error in the AVM value). However, in the absence of censoring, the positive errors and negative errors would approximately average out.



70. In the presence of censoring, AVM values that are above  $V$  (in the shaded area) are replaced by  $V$ . Thus, with censoring there are no longer any positive errors to balance out the negative errors, which means that the average value used to recalculate the LTV ratio will be less than  $V$ . The recalculated LTV ratio is then greater than the true value.

71. The above explanation gives an intuitive explanation of Dr. Kilpatrick's censoring bias, but that bias can also be calculated mathematically. Let  $A^*$  be the Greenfield AVM value, and assume that  $A^*$  is normally distributed with a mean equal to  $V$  and a standard deviation of  $\sigma$ . The value Dr. Kilpatrick uses to recalculate the LTV ratio (call it  $A$ ) is equal to  $V$  if  $A^*$  is greater than or equal to  $V$ , or  $A^*$  if  $A^*$  is less than  $V$ . The expected value (average) of  $A$  can then be calculated as follows:<sup>75</sup>

$$E[A] = Prob(A^* \geq V) * V + Prob(A^* < V) * E[A^* | A^* < V]$$

$$E[A] = 0.5 * V + 0.5 * \left( V + \sigma \frac{-\varphi(0)}{\Phi(0)} \right)$$

$$E[A] = V - \sigma \varphi(0) < V$$

72. In words, this calculation means that given Dr. Kilpatrick's methodology, the expected value (average) of the value used to recalculate the LTV ratio ( $E[A]$ ) is equal to the value originally used to calculate the LTV ratio ( $V$ ) minus the product of two numbers that are both greater than zero, which means that the expected value of the value used to recalculate the LTV ratio is less than the value originally used to calculate the LTV ratio. Thus, even if there were no difference on average between the Greenfield AVM value and the value originally used to calculate the LTV ratio, Dr. Kilpatrick's recalculated LTV ratios would be higher due to censoring bias.

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<sup>75</sup> This calculation is based on the formulas in W. Greene, *Econometric Analysis*, 5<sup>th</sup> ed., 2003, pp. 759-763.  $\Phi(0)$  is the cumulative distribution function of the standard normal distribution evaluated at zero.  $\Phi(0)$  is equal to 0.5 and hence cancels out between the second and third equations.  $\varphi(0)$  is the probability density function of the standard normal distribution evaluated at zero and is equal to approximately 0.4.

73. I now demonstrate that when Dr. Kilpatrick's errors are corrected, the recalculated LTV ratios are on average the same as the originally reported LTV ratios. Because the omitted variable correction discussed above can be calculated only for a subset of the loans in Dr. Kilpatrick's analysis, in Defense Exhibit 2808 ("DX-2808"), I correct only for Dr. Kilpatrick's prediction error (discussed above) and for his censoring bias. Column [C] of DX-2808 shows the average original LTV ratios for each supporting loan group, and column [D] shows the average based on Dr. Kilpatrick's recalculation. The difference between Dr. Kilpatrick's recalculation and the original LTV ratio is shown in column [E]. As DX-2808 shows, the average difference is 10.16%. In column [F] of DX-2808, I recalculate the LTV ratio based on a Greenfield AVM value that is corrected for Dr. Kilpatrick's prediction error, but continue to use Dr. Kilpatrick's methodology for calculating the LTV ratio, so censoring bias is still present. The average difference between the recalculated LTV ratio and the original LTV ratio correcting for Dr. Kilpatrick's prediction error but not for censoring bias is 6.53%, as shown in column [G]. In column [H] of DX-2808, I recalculate the LTV ratio based on a Greenfield AVM value that is corrected for Dr. Kilpatrick's prediction error and also correct for censoring bias by using only the AVM value in the denominator of the calculation. As column [I] shows, when both of these corrections are made the average difference between the recalculated LTV ratio and the original LTV ratio is 0.01%. The corrected recalculated LTV ratio is lower than the original LTV ratio for three of the seven supporting loan groups, further confirming the unbiased outcome.

74. In Defense Exhibit 2809 ("DX-2809"), I recalculate the LTV ratios including the omitted variable correction in addition to the prediction error and censoring bias corrections. Including the omitted variable correction results in a smaller sample size (an average of 61 loans

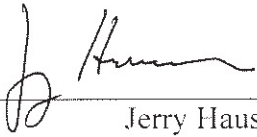
per supporting loan group compared to 95.9 in DX-2809), but leads to the same conclusion. For this set of loans, the average difference between Dr. Kilpatrick's recalculation and the original LTV ratio is 10.04%, as shown in column [E]. Incorporating the prediction error and omitted variable corrections reduces the average difference to 6.81%, as shown in column [G]. When censoring bias is also corrected for, the average difference is 0.79%, as shown in column [I]. The corrected recalculated LTV ratio is lower than the original LTV ratio for three of the seven supporting loan groups, further confirming the unbiased outcome.<sup>76</sup>

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<sup>76</sup> As a sensitivity analysis, I have also performed the calculations in DX-2808 and DX-2809 when transactions with sale prices or tax assessed values that are less than \$10,000 or greater than \$3,000,000 are removed from the calculation of the standard errors. This analysis, which involved removing 1% of total transactions, likely overstates the recalculated LTV ratios because it likely removes many valid transactions in addition to true outliers. Only transactions that have been investigated and found to reflect unreliable data (an investigation that Dr. Kilpatrick has not performed) should be excluded from the analysis. But even when a broader set of transactions, those with sale prices or tax assessed values that are less than \$10,000 or greater than \$3,000,000, are excluded from the calculation, the average difference between the recalculated LTV ratio and original LTV ratio is 2.18% in DX-2808 and 2.68% in DX-2809, with the corrected recalculated LTV ratio lower than the original LTV ratio for 2 of the 7 supporting loan groups in DX-2808 and 3 of the 7 supporting loan groups in DX-2809. Thus, this analysis demonstrates that Dr. Kilpatrick's recalculated LTV ratios are substantially overstated even when such transactions are removed. Dr. Kilpatrick neither investigated the reliability of the data excluded by his filters, nor performed a similar sensitivity analysis for any of those filters.

## VII. CONCLUSION

75. It is my opinion, based on the extensive analyses I have performed, that the Greenfield AVM is fundamentally unreliable and inaccurate. When the serious errors committed by Dr. Kilpatrick in constructing and applying the Greenfield AVM are corrected, Dr. Kilpatrick provides no evidence of any overstatement in the appraised values of the Nomura Sample Loans.

  
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Jerry Hausman

SWORN to before me  
this 19<sup>th</sup> day of February 2015

  
\_\_\_\_\_  
Notary Public



**THERESA CZARCINSKI**  
**Notary Public**  
**Commonwealth of Massachusetts**  
**My Commission Expires**  
**September 29, 2017**